A Strategy for Simplyfing Multimode Weighted Network Data Structure

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In recent years, it has become more frequent the use of complex network data structures to capture the richness of social phenomena. Multipartite networks are an example where a multitude of scenarios are represented with different types of relations, actors or modes (e.g., multilayer, multiplex, multilevel, multimode networks, and various temporal snapshots in a longitudinal perspective). Starting from these data structures, the present contribution discusses an analytic strategy to simplify multipartite weighted networks in which sets of different units are connected.

To the best of our knowledge, few papers are devoted to the analysis of multimode networks representing a generalization of the conceptual basis and the matrix formalism of two-mode networks and bipartite graphs. In the seminal paper of Fararo and Doreian [1], a tripartite network consists of three types of nodes and ties are present only between nodes of distinct types. Such a structure can be extended to any number of modes, given rise to multimode networks examining the collection of all two-mode networks [2].

Formally, a multimode network $\mathcal{M}$ can be conceived as consisting of a pair $(\mathcal{V}, \mathcal{E})$, being $\mathcal{V} = \{V_i\}_{i=1,\ldots,K}$ the collection of K set of nodes, one for each mode i, $V_i \cap V_j = \emptyset$, $\forall i \neq j$, and being $\mathcal{E} = \{E_{ij}\}_{(i,j=1,\ldots,K)}$, $E_{ij} \subseteq V_i \times V_j$, $\forall i \neq j$ $E_{ii} = \emptyset$, $\forall i$, the collection of edges existing among the nodes belonging to the i-th and j-th mode. As said, the multimode network can be seen as the collection of all possible two-mode networks $\mathcal{G}_{ij} = (V_i, V_j, E_{ij})$ among the K set of nodes. Following the original idea of tripartite graphs, given the multimode network $\mathcal{M}$, a unique adjacency matrix $\mathbf{A}$ can be defined given by the combination in a block matrix of the sociomatrix $\mathbf{A}_{ij}$ corresponding to the two-mode networks $\mathcal{G}_{ij}$.

Such complex data structures may arise in different application fields, like in folksonomy (users, texts, tags, topics, etc.), in bibliographic data (papers, journals, keywords, references, etc.), in genomic networks (genes, diseases, patients, etc.). Moving from these real-data scenarios, here we explore intra-national student mobility, thanks to the availability of data at individual level extracted from the Italian MOBYSU.IT database. From this archive, *multimode weighted networks* are formally investigated and analyzed. In our case, we consider a multimode network with five modes: $\mathcal{V}^1 \equiv \mathcal{S} \equiv \{s_1, \ldots, s_i, \ldots, s_I\}$ the set of students; $\mathcal{V}^2 \equiv \mathcal{U} \equiv \{u_1, \ldots, u_j, \ldots, u_J\}$ the set of universities; $\mathcal{V}^3 \equiv \mathcal{E} \equiv \{e_1, \ldots, e_k, \ldots, e_K\}$ the set of educational programmes; $\mathcal{V}^4 \equiv \mathcal{R} \equiv \{r_1, \ldots, r_h, \ldots, r_H\}$ the set of regions; and $\mathcal{V}^5 \equiv \mathcal{P} \equiv \{p_1, \ldots, p_c, \ldots, p_C\}$ the set of provinces.

In the present paper we explore the relation of multimode networks and hypergraphs, and we propose a strategy to analyze these complex networks based on simplification, normalization, and filtering procedures. In particular, we introduce a model-based approach for bipartite networks weighted in order to extract statistically significant links.

Keywords: Multimode network, Network simplification, Intra-national mobility, Tertiary education

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