Relationships Among Nonrandom/Random Score Formulations of PCA and Factor Analysis

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The purpose of this study is to show hierarchical relationships among some formulations of PCA and factor analysis (FA). The relationships follow from a comprehensive FA (CompFA) model [1], in which a multivariate observation is expressed as the sum of common factor, specific factor, and error parts. This model is divided into the nonrandom (N-CompFA) and random (R-CompFA) ones, with the factor parts treated as nonrandom in the former, but as random in the latter.

First, it is discussed how the N-CompFA model implies the following hierarchy; matrix-decomposition FA (MDFA) > completely decomposed FA (CDFA) [2] > PCA. Here, the procedure after > is a constrained variant of the one before >.

Then, the following three facts are proved: (a) Minimum rank FA (MRFA) [3] can be regarded as a procedure for the R-CompFA model; (b) By constraining the specific factor part to be null in MRFA, it leads to random-version of PCA (RPCA); (c) By constraining the error part to be uncorrelated among variables in RPCA, it leads to the prevalent FA (PrevFA) procedure. These facts and PrevFA > probabilistic PCA (PPCA) [4] lead to hierarchy MRFA > RPCA > PrevFA > PPCA.

Finally, it is shown that the above two hierarchies can be unified as MDFA > CDFA/MRFA > PCA/RPCA > PrevFA > PPCA, using the equivalence of the CDFA solution to the MRFA one and that of the PCA solution to the RPCA one.

Key words : CompFA model, MDRA, CDFA, MRFA, RPCA

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