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Predictive biplots for individual differences scaling (INDSCAL) models

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Customarily, individual differences scaling (INDSCAL) models deal with two mode, three way data. The typical input format is a set of K distance matrices each of size $n \times n$ originating from, for instance, K judges who rate the differences between n items. Judge k rates each item on p variables and provides its ratings in an $n \times p$ matrix, say $\boldsymbol{X}_k$, from which the set of distance matrices is derived. Parallel to classical scaling, also known as Principal Coordinate Analysis, a set of positive semi-definite symmetric matrices are formed by double centring the squared distance matrices. In general, for any set of K positive semi-definite symmetric matrices, the INDSCAL model finds the best, in the least squares sense, representation of the n items in r, usually 2, dimensions and an associated set of r weights for each of the K judges. Hence, INDSCAL models consider symmetric matrices $\boldsymbol{B}_k = \boldsymbol{X}\boldsymbol{W}_k\boldsymbol{X}'$ for $k = 1, \dots, K$, where $\boldsymbol{X} : n \times r$ is a compromise judges matrix termed the group-average and $\boldsymbol{W}_k$ is a diagonal matrix of weights given by the kth judge to the r, specified in advance, columns of $\boldsymbol{X}$; non-negative weights are preferred and usually $r < n$. The basic INDSCAL criterion to be minimized is: $min \sum_{k=1}^{K} \|\boldsymbol{B}_k - \boldsymbol{X}\boldsymbol{W}_k\boldsymbol{X}'\|^2$ where the minimization is over $\boldsymbol{X}$ and the $\boldsymbol{W}_k$. A major issue is that simultaneous least-squares estimates of the parameters may be found without imposing constraints. However, group average and individual judge weighting parameters may not be estimated uniquely, without imposing some subjective constraint. We follow the recommendation of [1] by using linear constraints $\sum_{k=1}^{K} \mathbf{1}'\boldsymbol{W}_k = \mathbf{1}'$, as it enables a comparison of the weights obtained (i) within group k and (ii) between the same item drawn from two or more groups. In the example that follows we have implemented the two-phase ALS algorithm proposed by [1]. It (i) computes for fixed $\boldsymbol{X} : n \times r$ the weights $\boldsymbol{W}_k$ subject to $\sum_{k=1}^{K} \mathbf{1}'\boldsymbol{W}_k = \mathbf{1}'$, and then (ii) keeping $\boldsymbol{W}_k$ fixed, it updates $\boldsymbol{X}$. These two steps are iterated until convergence.

Traditionally, two plots can be made for $r = 2$, viz. the subjects (judges) space, based on the K sets of weights and the group stimulus space, representing the n objects/items. Our aim is to introduce some enhancements to these plots. Assuming that the dissimilarities between the objects/items were generated by observations on p variables, we now want to simultaneously represent the n items and p variables in a biplot. In particular, the variables are to be represented as calibrated axes that allow inferring the values of any point (object) for all variables i.e. to predict the value of any given point for any of the variables. However, this is not straight forward as the input distance matrices do not contain information about the individual p variables. In this paper we will illustrate how to represent the variables with the objects in the group stimulus space by considering some suggestions made by [2] for general multidimensional scaling biplots. We implement the following methods for constructing prediction biplots in an example: coherent prediction and two incoherent approximate prediction methods viz. (i) Procrustanean prediction and (ii) regression prediction. Furthermore, we extend the resulting biplots by adding information of the individual judges using interpolation. Representing the variables as biplot axes, allows for the prediction of the values for each of the p variables for any point in the r-dimensional biplot space. Interpolation is the opposite, namely finding the r-dimensional coordinates for new observations on the p variables. This enables us to interpolate the judges into our predictive INDSCAL biplot, resulting in a more informative visual display of the data.

Keywords: Biplots, Individual Differences Scaling, Multidimensional Scaling, Procrustean Prediction, Regression Prediction

References

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