Simultaneous Orthogonal Rotation of Parameter Matrices in Generalized Structured Component Analysis

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Generalized structured component analysis (GSCA) is a component-based structured equation modeling (SEM) procedure that considers the interactions of multiple observed variables using multiple latent variables [1, 2]. In GSCA, latent variables are defined as a linear combination of the observed variables, and GSCA is therefore referred to as a component-based SEM procedure. GSCA is defined as a minimization of $f_{GSCA}(\mathbf{W}, \mathbf{C}, \mathbf{B}) = ||\mathbf{Z} - \mathbf{ZWC}||^2 + ||\mathbf{ZW} - \mathbf{ZWB}||^2$, where $\mathbf{Z}(N \times J)$ denotes a data matrix, and $\mathbf{W}(J \times P)$, $\mathbf{C}(P \times J)$, $\mathbf{B}(P \times P)$ are the parameter matrices to be estimated.

Inspired by Exploratory SEM [3] which explores simple relationships between observed/latent variables based on factor-based SEM procedure, the research first shows the rotational indeterminacy of parameter matrices in GSCA. Namely, we have $f_{GSCA}(\mathbf{W}, \mathbf{C}, \mathbf{B}) = f_{GSCA}(\mathbf{WS}', \mathbf{SC}, \mathbf{SBS}')$ with $\mathbf{S}'\mathbf{S} = \mathbf{I}$. The indeterminacy ensures that the all parameter matrices in GSCA are orthogonally rotated by $\mathbf{S}$ under some reasonable constraints. The research proposes a new rotation procedure that aims to simplify multiple parameter matrices simultaneously. The procedure is formulated as a minimization problem of a least squares criterion defined as a sum of Procrustes criteria; $l(\mathbf{S}, \mathbf{W}^\sharp, \mathbf{C}^\sharp, \mathbf{B}^\sharp) = (JP)^{-1}||\mathbf{W}^\sharp - \mathbf{WS}'||^2 + (JP)^{-1}||\mathbf{C}^\sharp - \mathbf{SC}||^2 + (P^2)^{-1}||\mathbf{B}^\sharp - \mathbf{SBS}'||^2$ where $\mathbf{W}^\sharp$, $\mathbf{C}^\sharp$, $\mathbf{B}^\sharp$ are the target matrices for $\mathbf{W}$, $\mathbf{C}$, $\mathbf{B}$ respectively. Exploratory GSCA (EGSCA) is newly introduced, which is defined as the sequential use of GSCA to obtain the initial solution, followed by their simultaneous orthogonal rotation.

The effectiveness of EGSCA was investigated by numerical simulation, where EGSCA was applied to a simulated dataset generated by the hypothetical model. The result showed that EGSCA recovers the true simple structure of the parameter matrices. Furthermore, EGSCA was applied to the real dataset analyzed by Hwang & Takane (2004) to demonstrate the original GSCA procedure. Importantly, the application showed that EGSCA proposed a better model than the one reported by Hwang & Takane (2004), indicating the benefit of GSCA for an exploratory purpose.

[1] H. Hwang and Y. Takane, "Generalized structured component analysis," *Psychometrika*, vol. 69, no. 1, pp. 81–99, 2004.

[2] H. Hwang and Y. Takane, *Generalized structured component analysis: A component-based approach to structural equation modeling.* CRC Press, 2014.

[3] T. Asparouhov and B. Muthén, "Exploratory structural equation modeling," *Structural equation modeling: a multidisciplinary journal*, vol. 16, no. 3, pp. 397–438, 2009.